Using Higher Moments of Fluctuations and their Ratios in the Search for the QCD Critical Point

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Misha Stephanov (University of Illinois)

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Outline

- Introduction
- Critical Contribution to Particle Multiplicity Fluctuations
- Ratios of Fluctuation Observables
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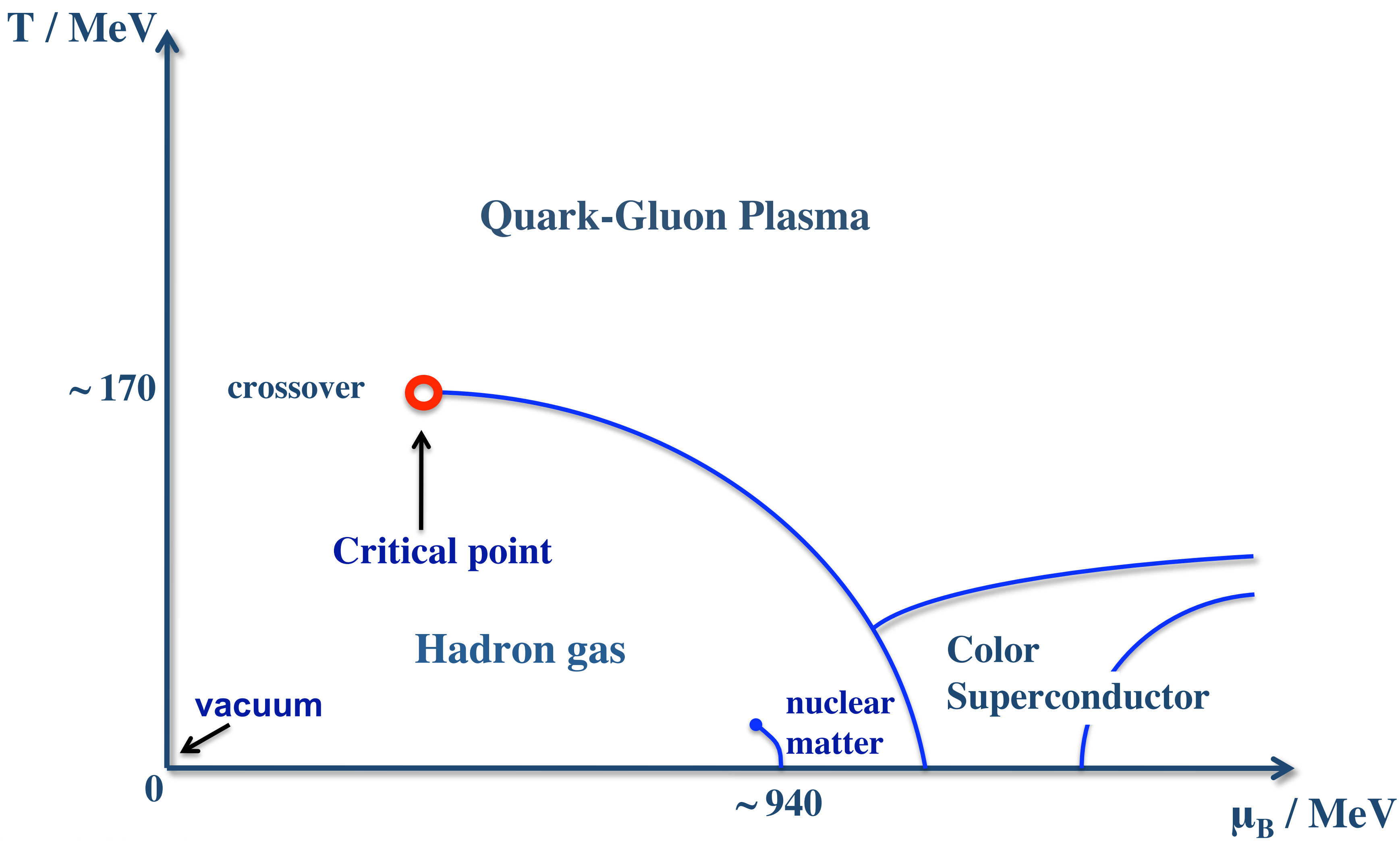

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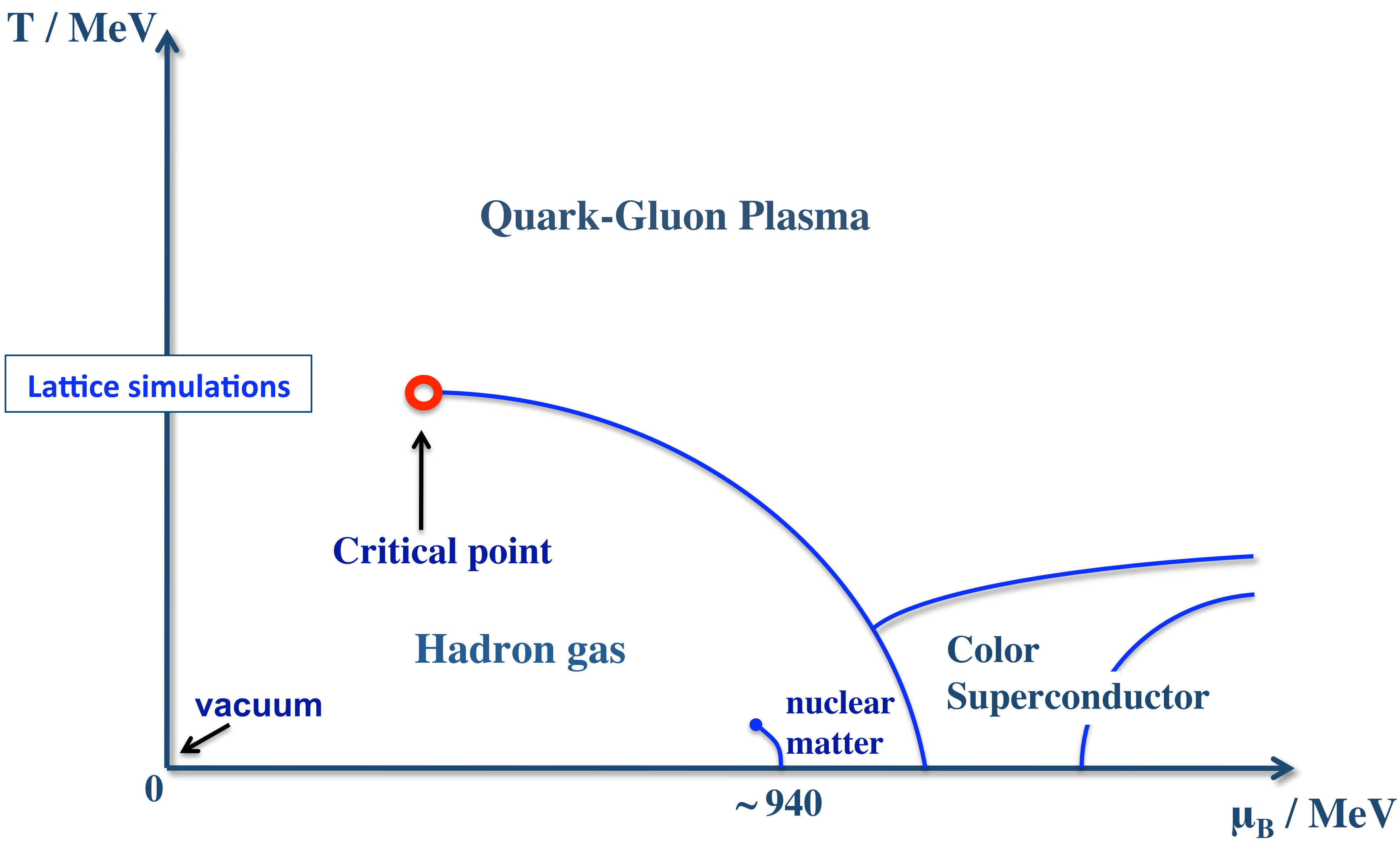

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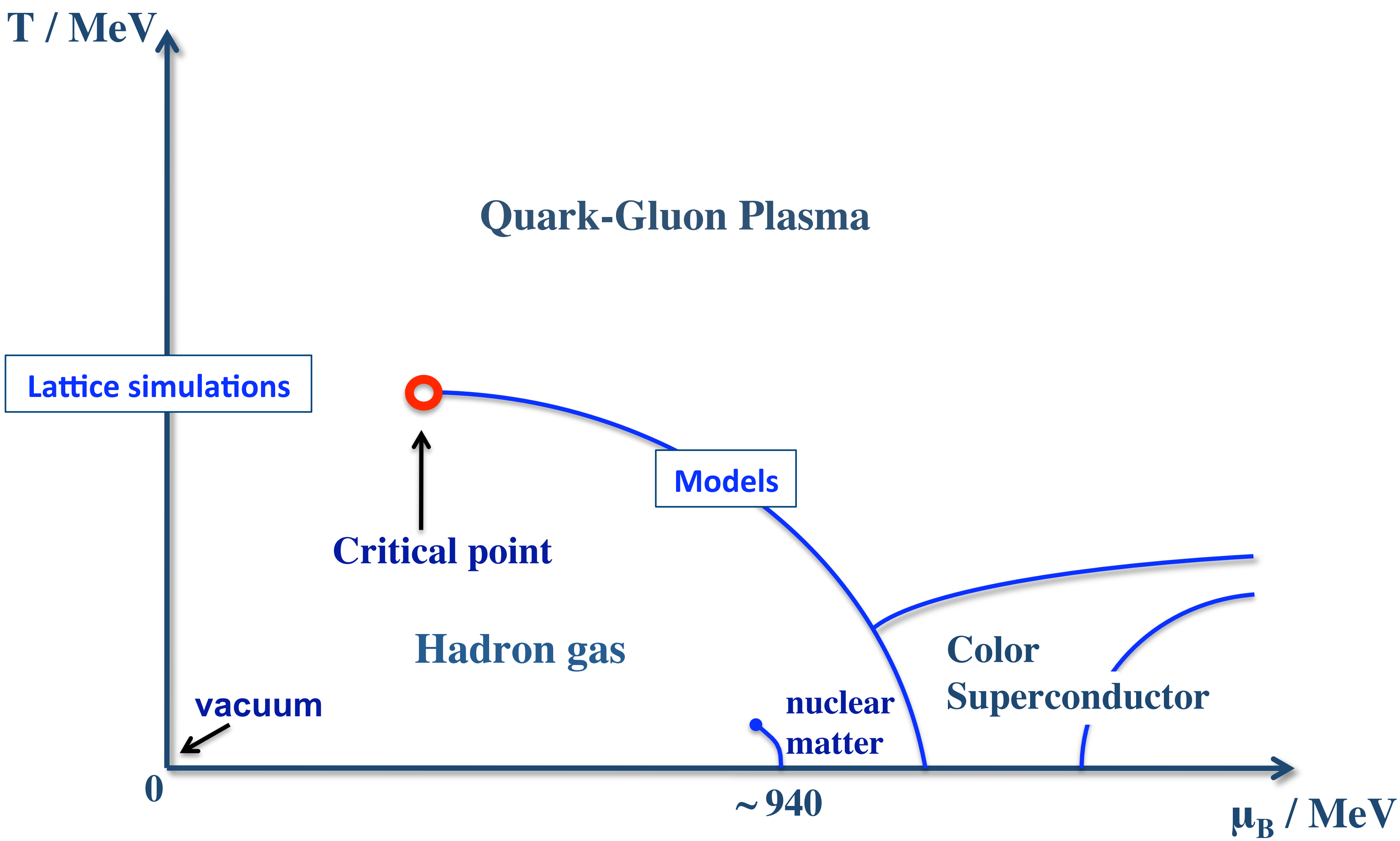

Heavy-Ion Collision Experiments

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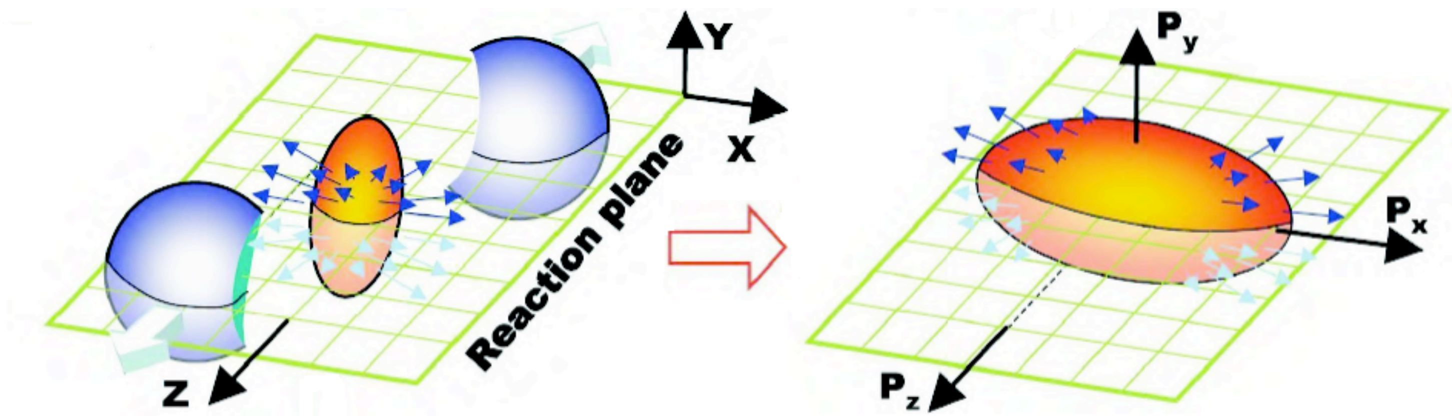
- RHIC: Au-Au collisions at $\sqrt{s_{\max}} = 200\ \mathrm{GeV}$
- Momentum asymmetry ➡ collective flow
  ➡ strongly-coupled QGP



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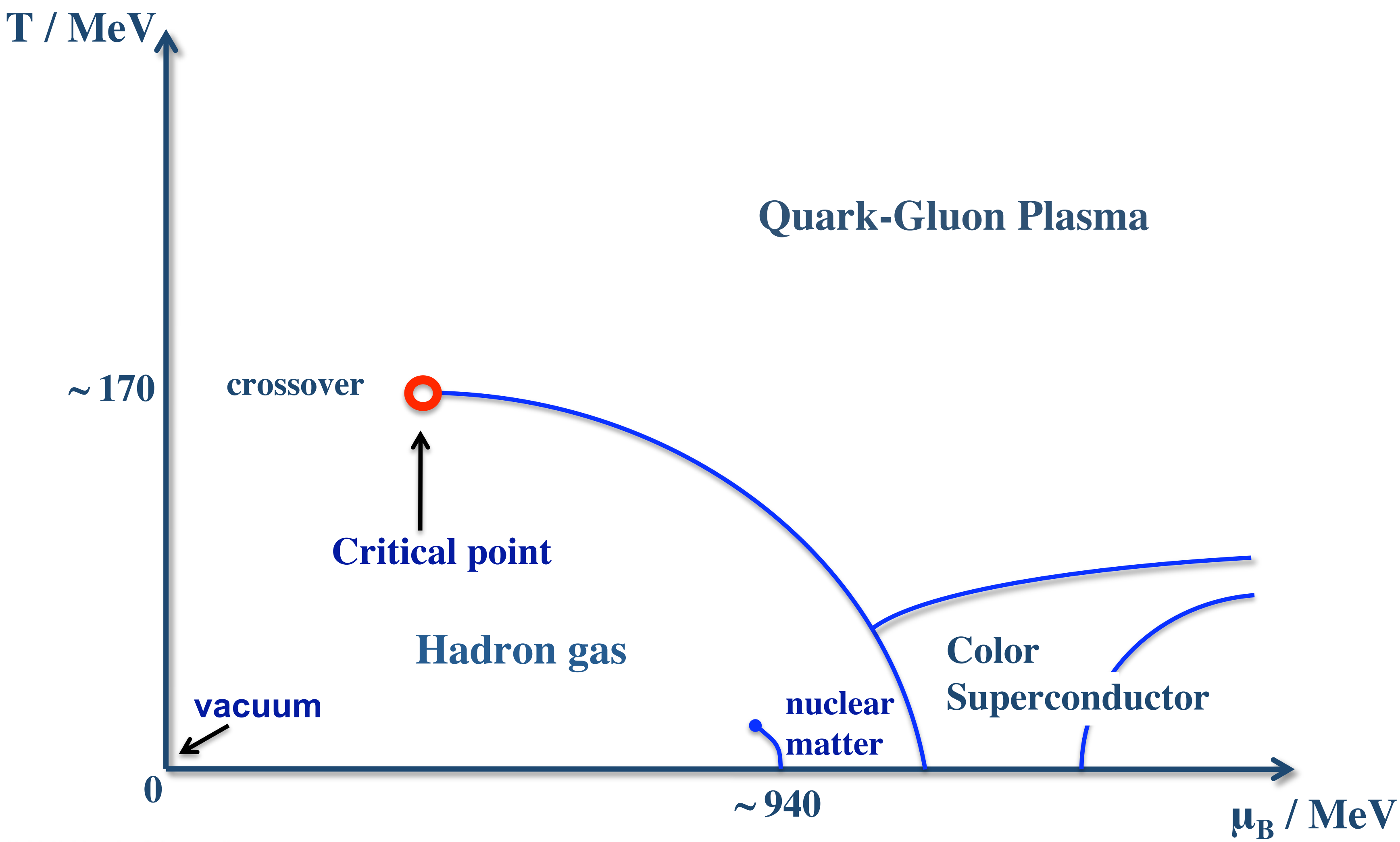

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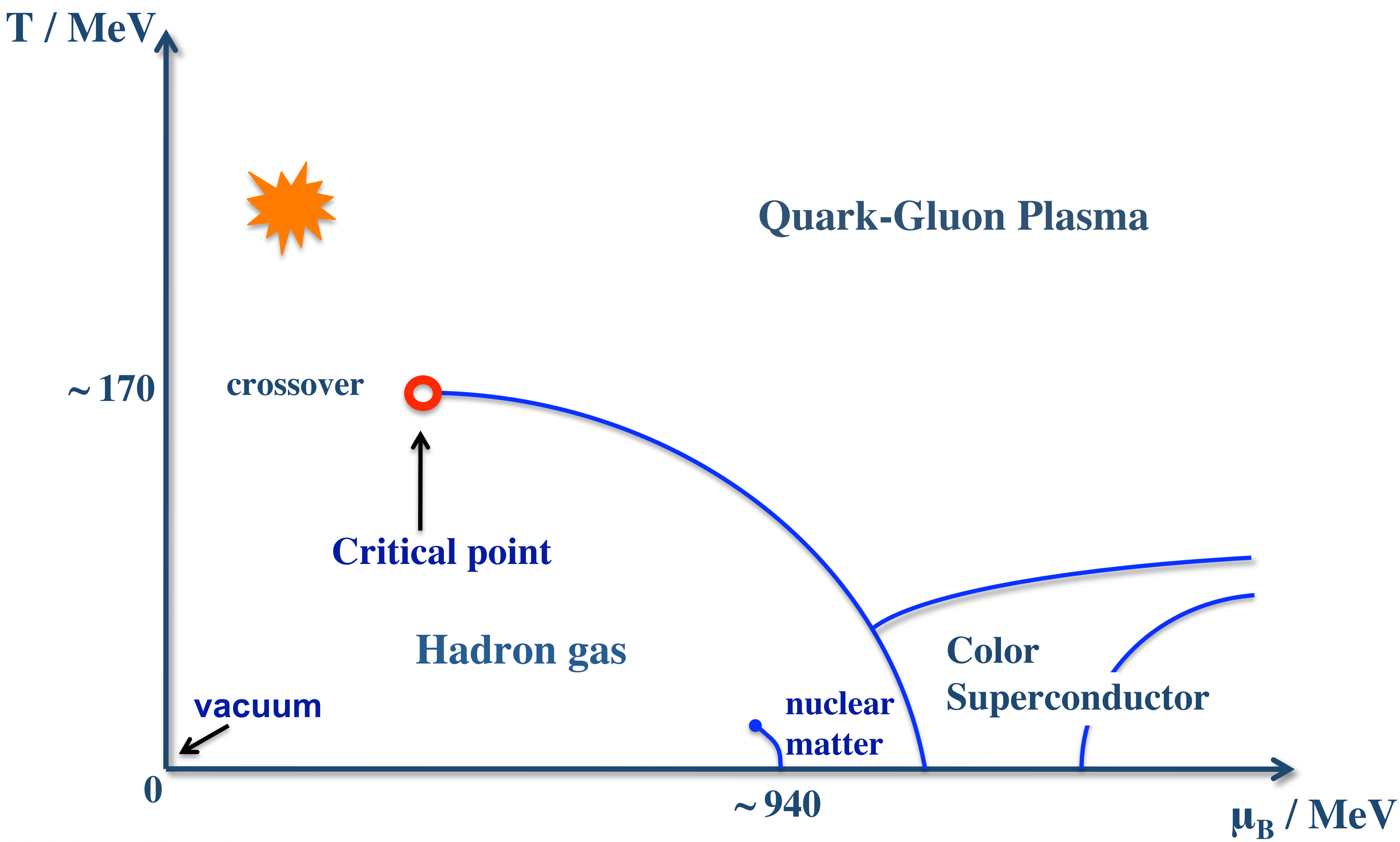

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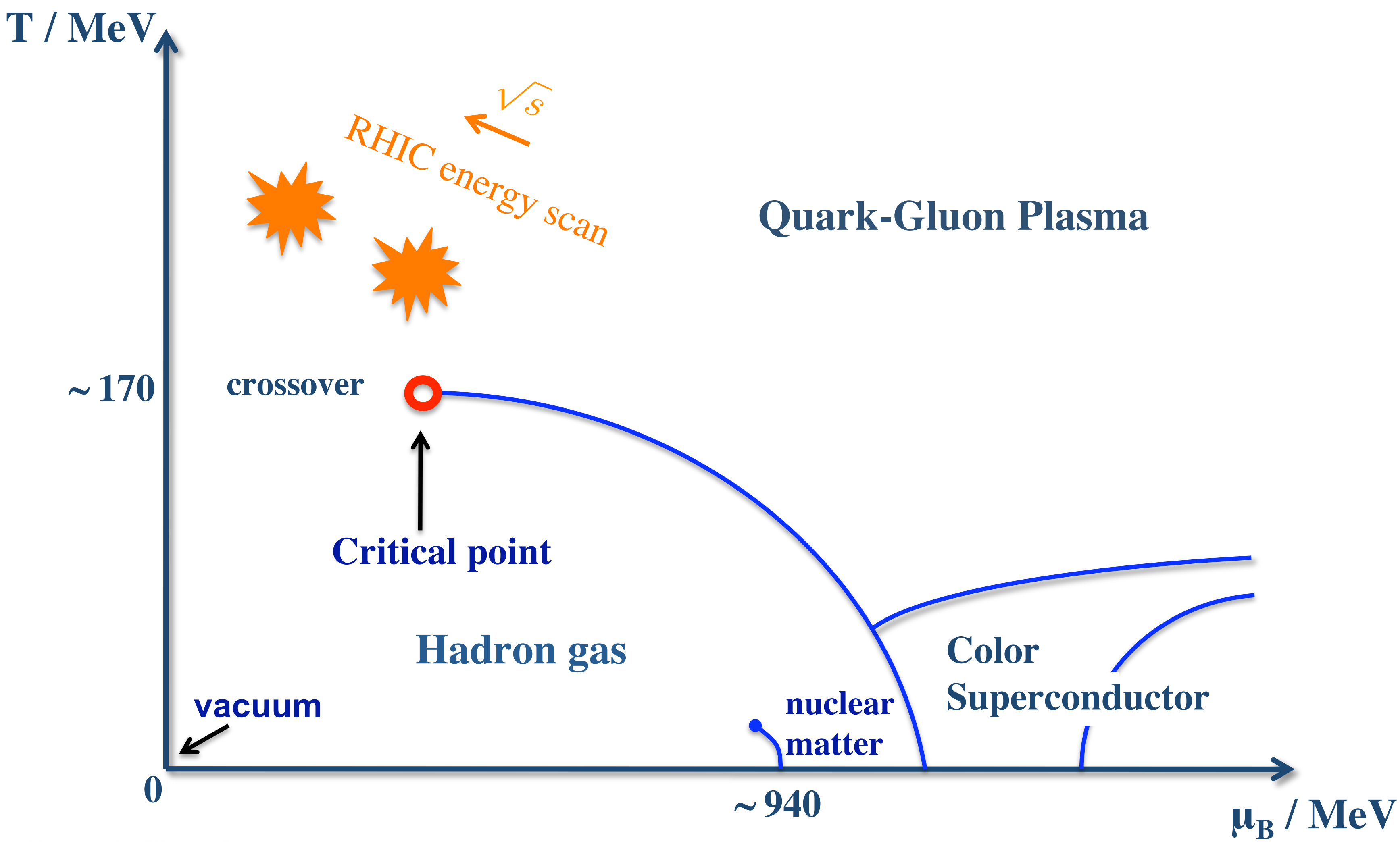

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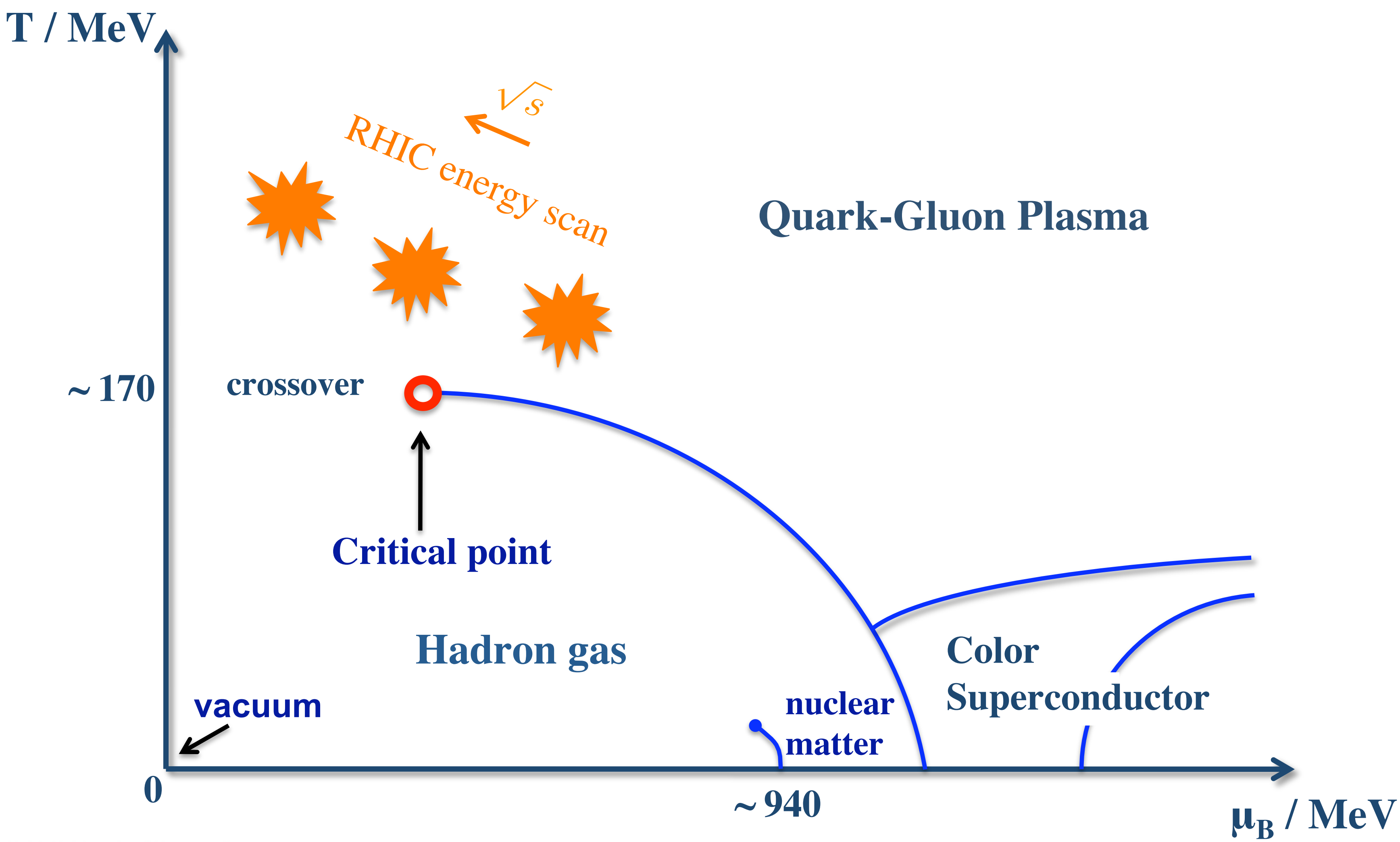

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Heavy-Ion Collision Experiments - continued

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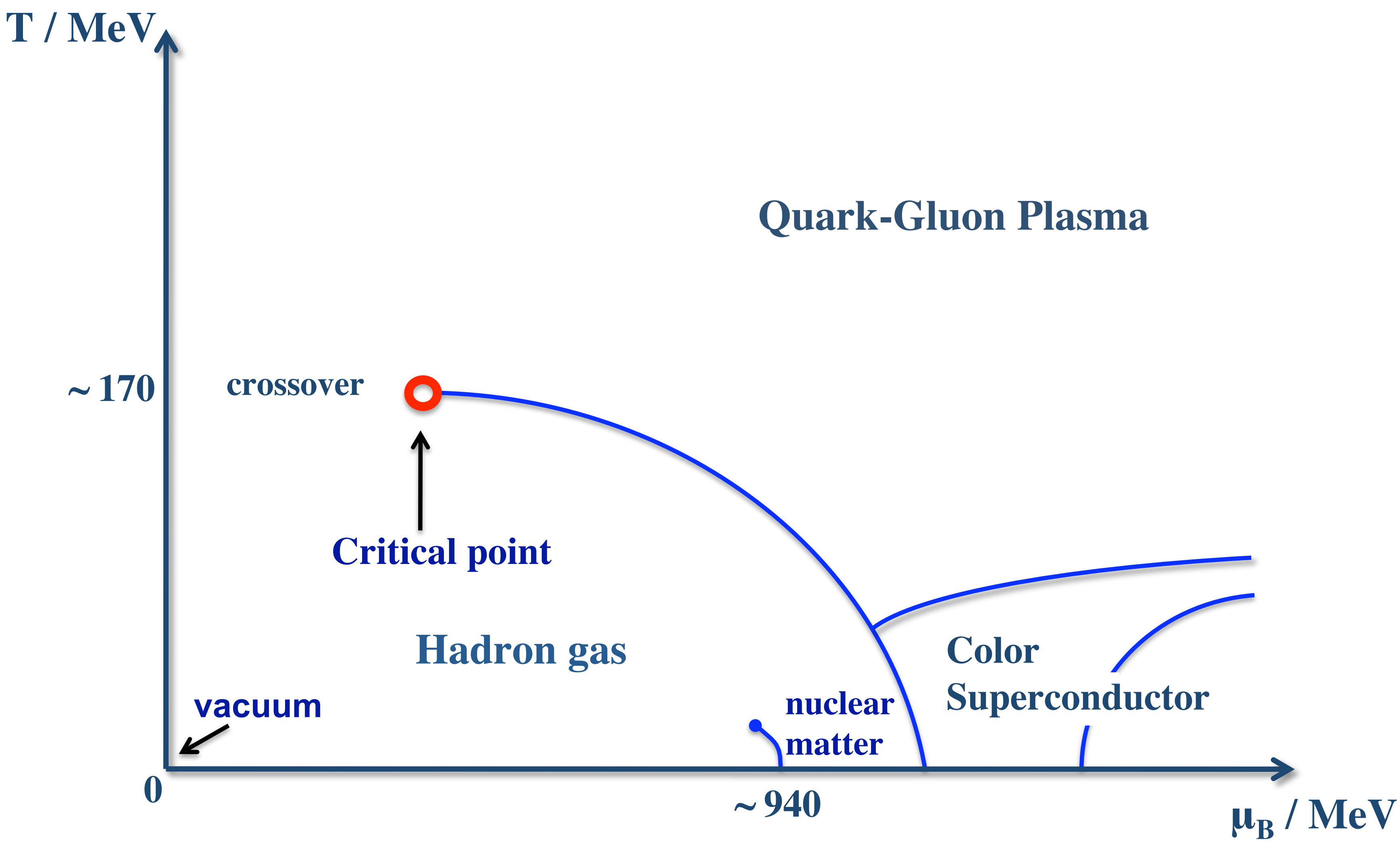

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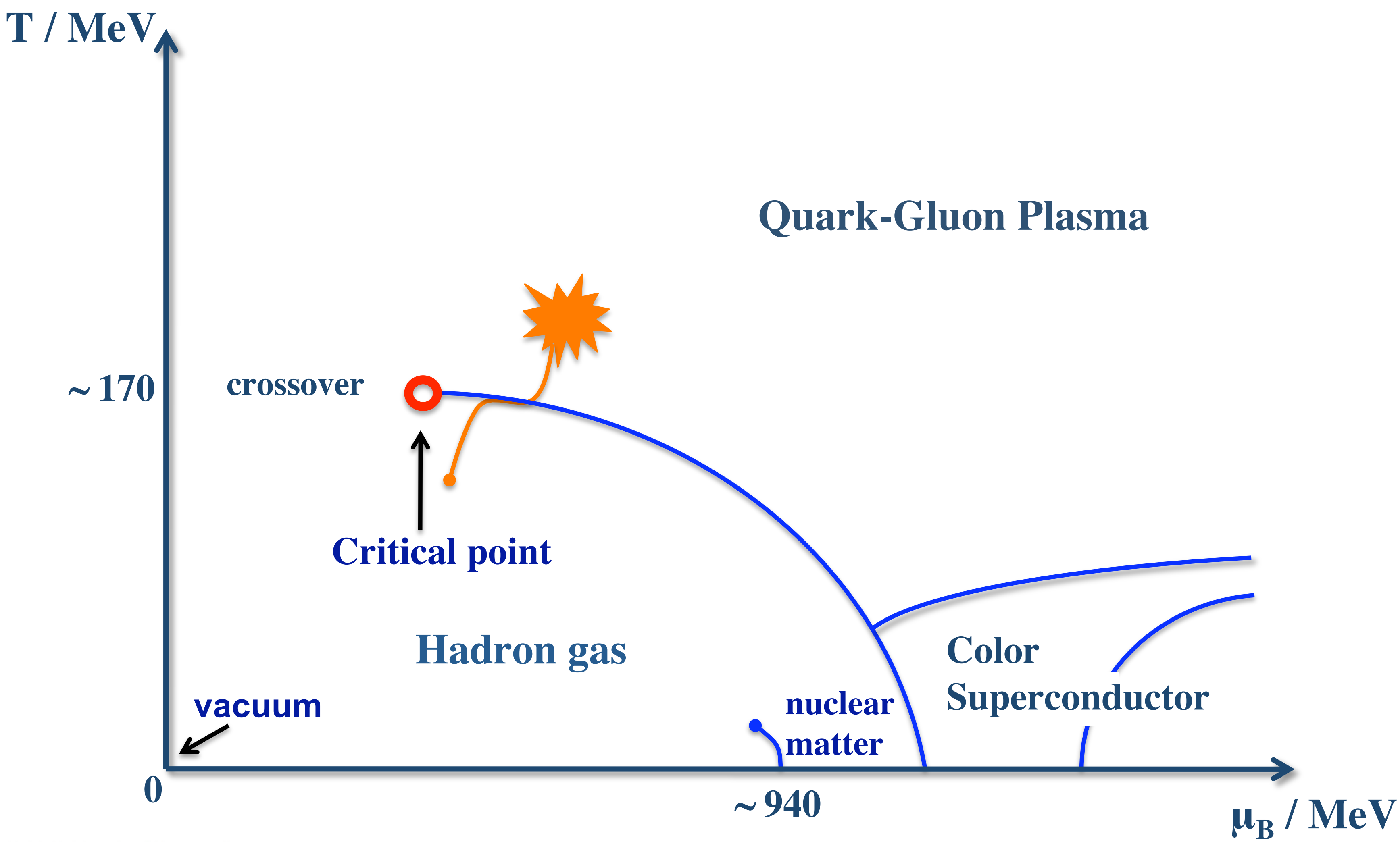

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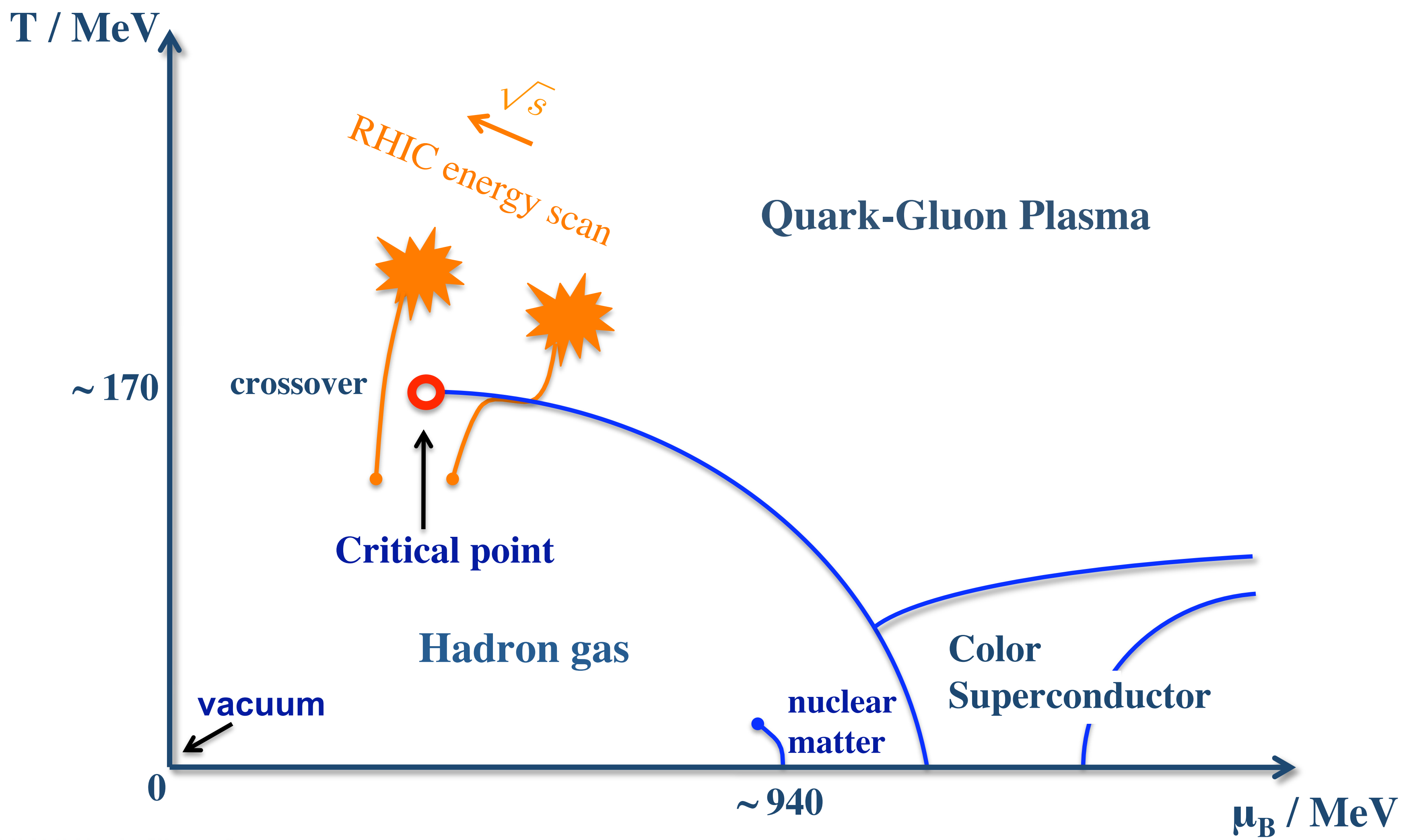

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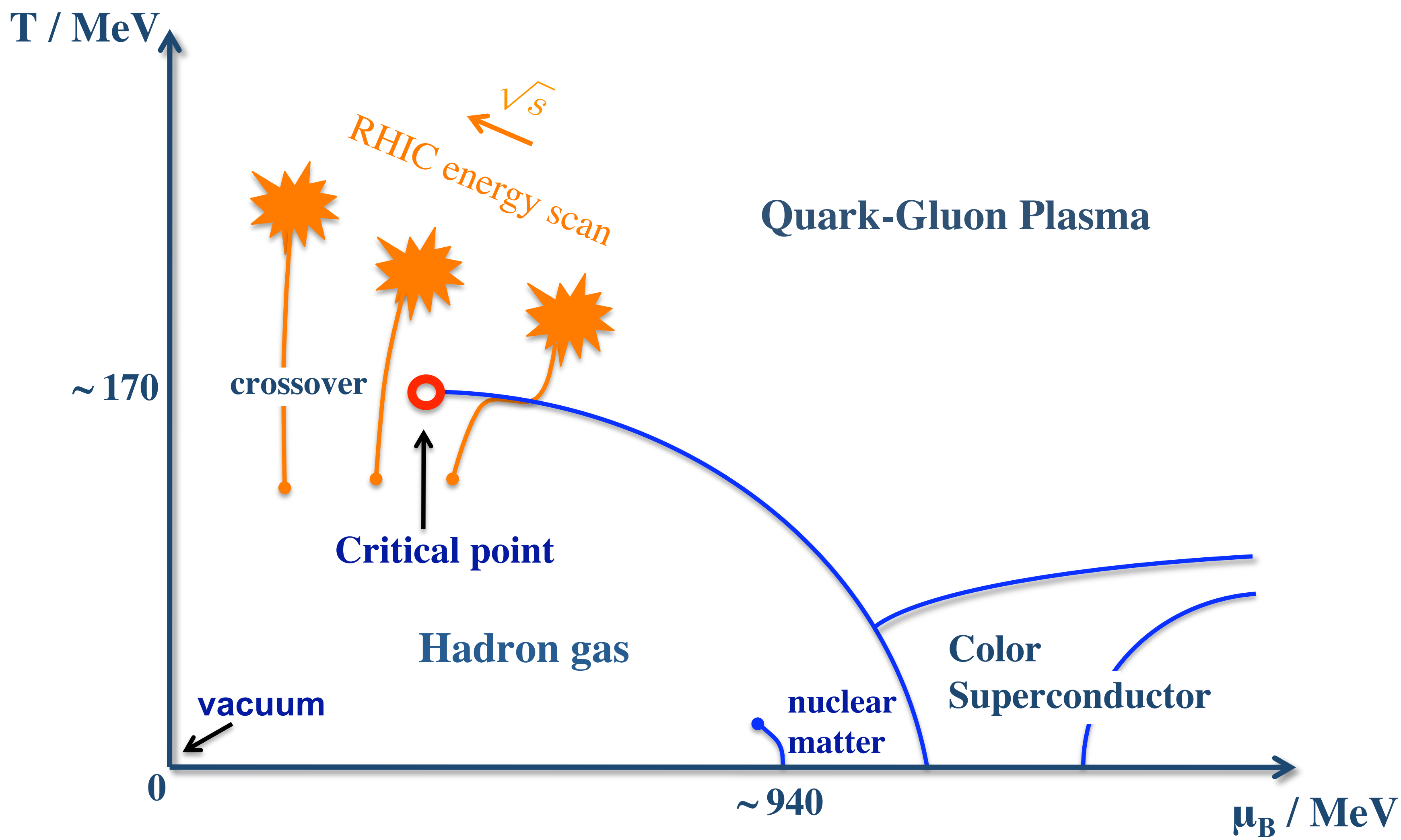

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→ Event-by-Event fluctuations

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$$\Omega(\sigma) = \int d^3x \left[\frac{1}{2}(\nabla\sigma)^2 + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4 + \ldots\right].$$

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- Correlation length $\xi = m_\sigma^{-1}$ diverges at the CP
- Near the CP: $\lambda_3 = \widetilde{\lambda}_3\, T\, (T\,\xi)^{-3/2},\ \ \lambda_4 = \widetilde{\lambda}_4\, (T\,\xi)^{-1}$ with $0 \lesssim \widetilde{\lambda}_3 \lesssim 8,\ \ 4 \lesssim \widetilde{\lambda}_4 \lesssim 20$ dimensionless and known in the Ising universality class

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- $\xi \to \infty$ at CP in the thermodynamic limit $(t, V \to \infty)$

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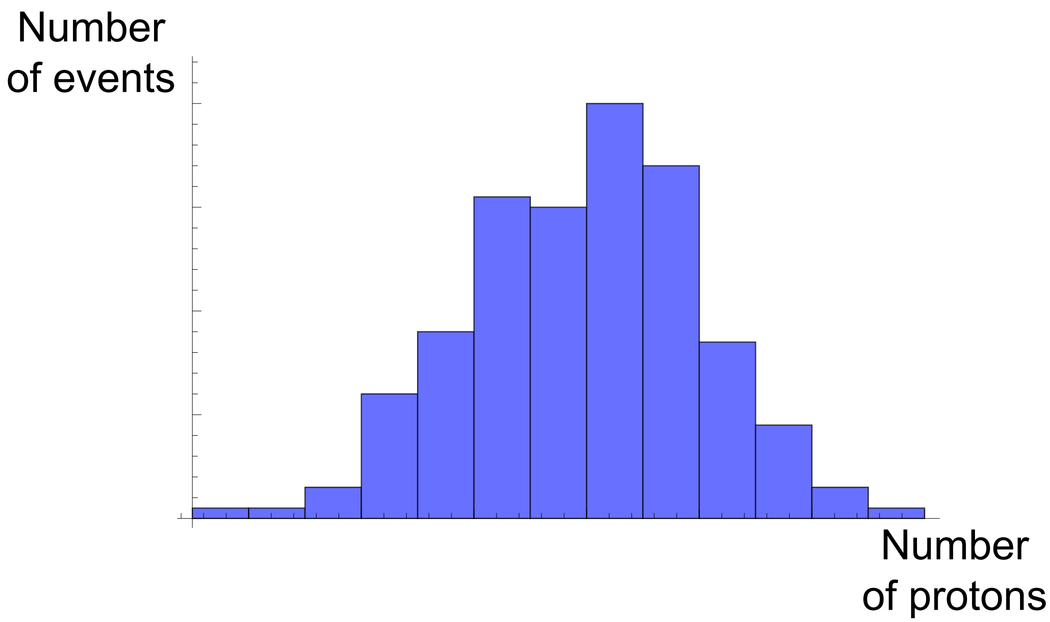

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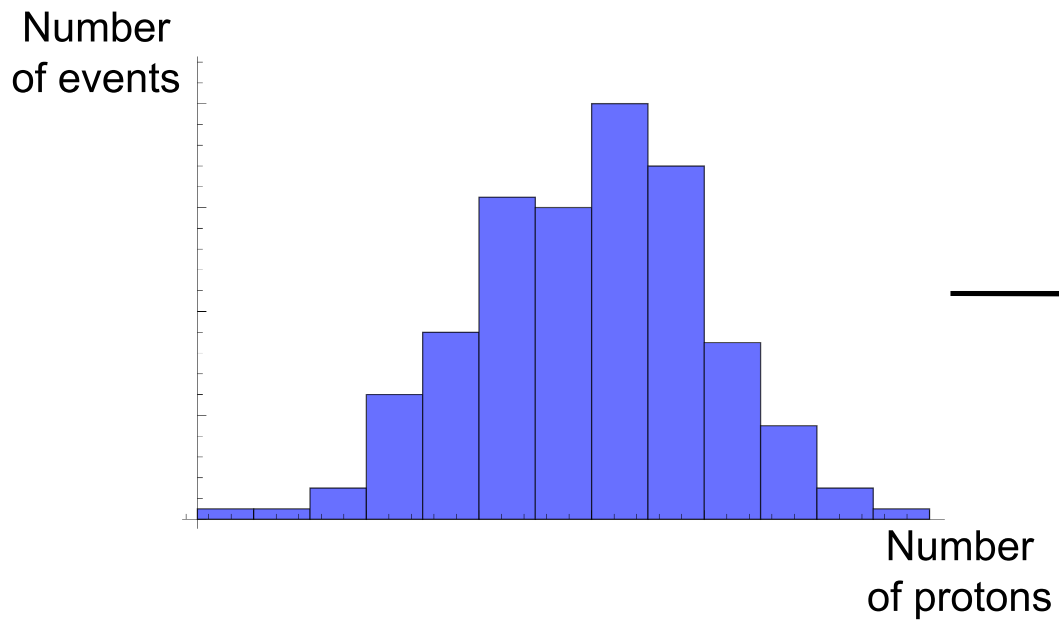

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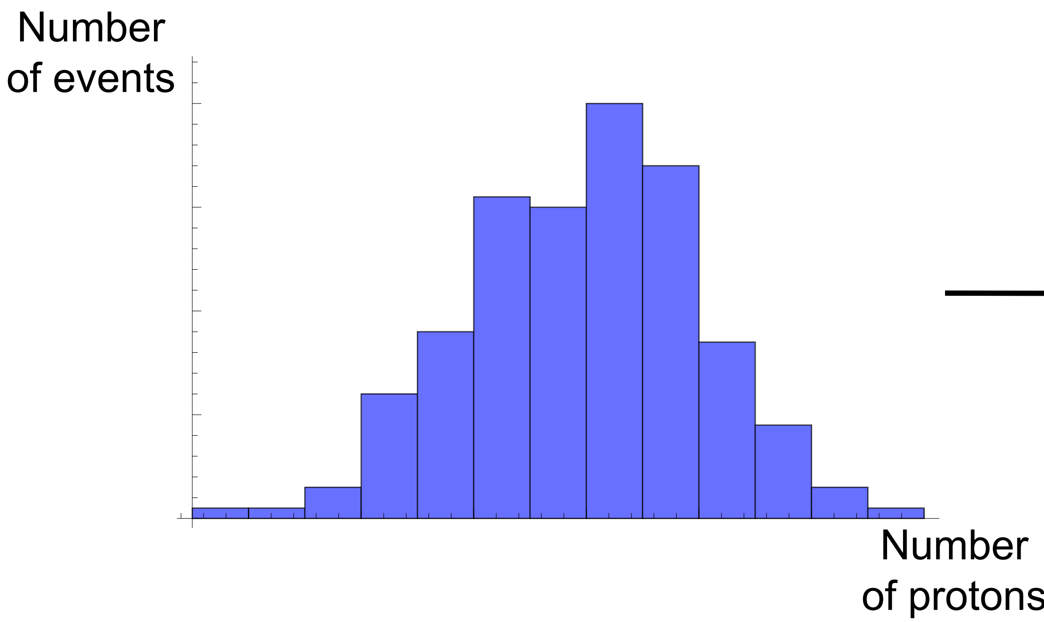

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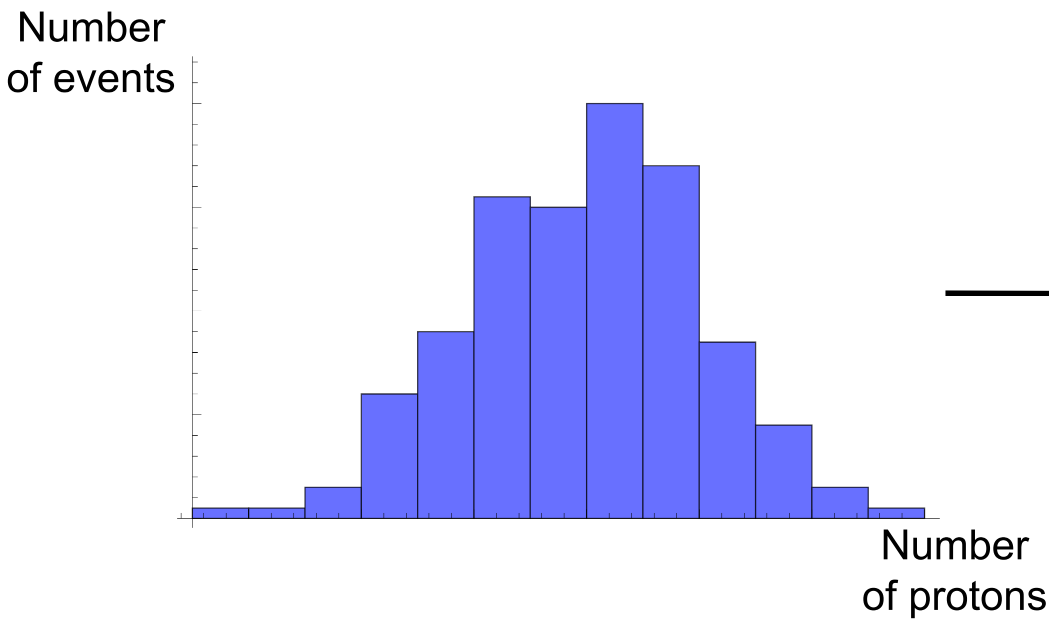

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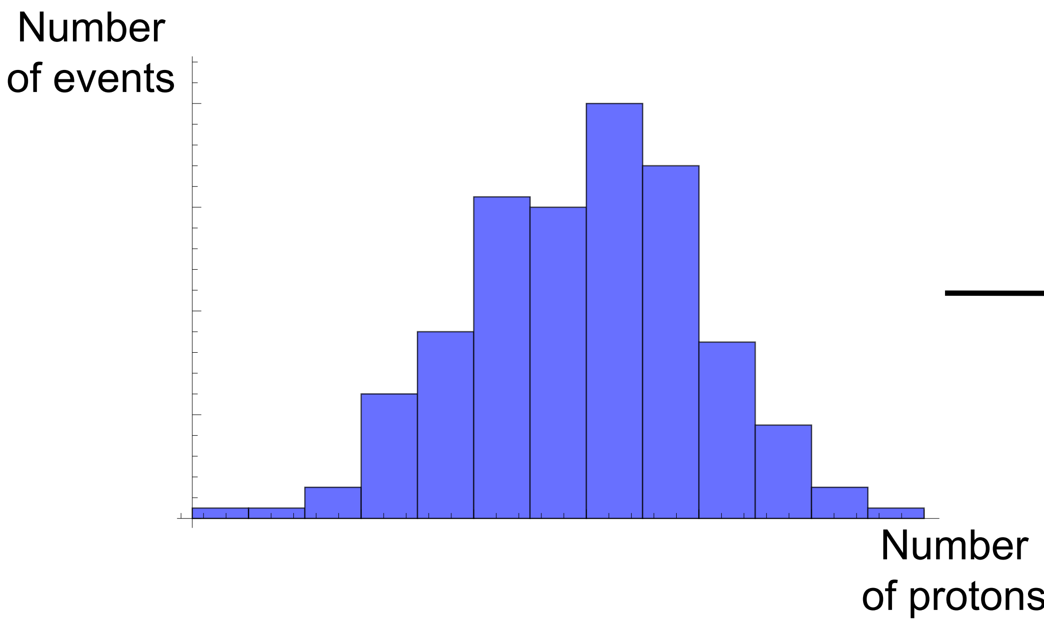

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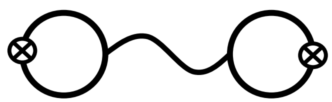
- We will use cumulants, e.g.:

$$\kappa_2 = \langle N^2 \rangle, \quad \kappa_3 = \langle N^3 \rangle, \quad \kappa_4 \equiv \langle\langle N^4 \rangle\rangle = \langle N^4 \rangle - 3\langle N^2 \rangle^2$$

Critical contribution to pion/proton correlators



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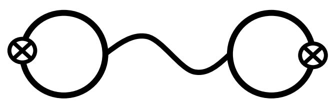

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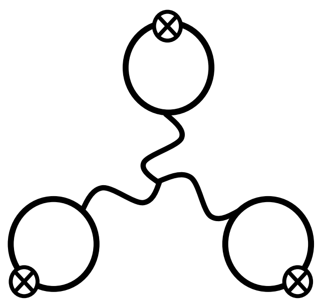

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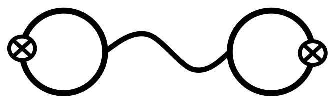

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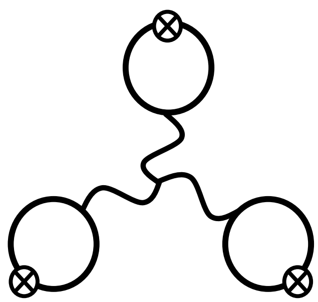

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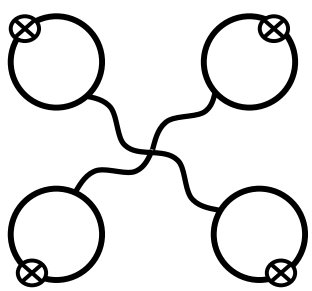

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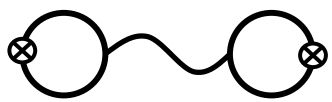

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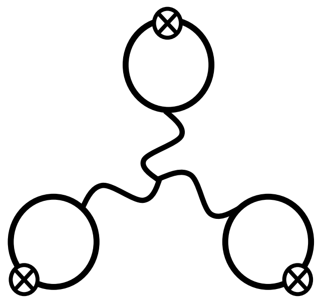

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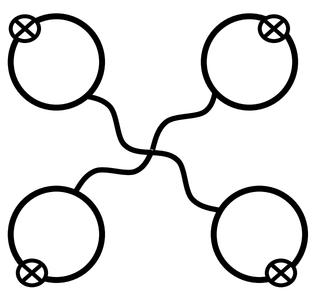

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  Adapt previous expressions by replacing:

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- Note: correlators depend on 5 parameters:

$$G,\; g,\; \xi,\; \tilde{\lambda}_3,\; \tilde{\lambda}_4$$

  which have large uncertainties

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Calculating multiplicity cumulants

- Second cumulant – variance:

$$\kappa_{2p,\sigma} = \langle (\delta N_p)^2 \rangle_\sigma = \int_{\mathbf{k_1}} \int_{\mathbf{k_2}} \langle \delta n_{\mathbf{k_1}} \delta n_{\mathbf{k_2}} \rangle_\sigma \propto V^1$$

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- For mixed cumulants with *i* protons and *j* pions:

$$\omega_{ipj\pi} = \frac{\kappa_{ipj\pi}}{N_p^{\frac{i}{i+j}} N_\pi^{\frac{j}{i+j}}}$$

Calculating multiplicity cumulants

- Second cumulant – variance:

$$\kappa_{2p,\sigma} = \langle (\delta N_p)^2 \rangle_\sigma = \int_{\mathbf{k_1}} \int_{\mathbf{k_2}} \langle \delta n_{\mathbf{k_1}} \delta n_{\mathbf{k_2}} \rangle_\sigma \propto V^1$$

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Poisson

-Bose-Einstein effects
-Resonance decays
-Etc..

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Multiplicity cumulants – critical point signature

- Higher cumulants depend stronger on ξ:

$$\omega_2 \propto \xi^2, \quad \omega_3 \propto \xi^{9/2}, \quad \omega_4 \propto \xi^7$$

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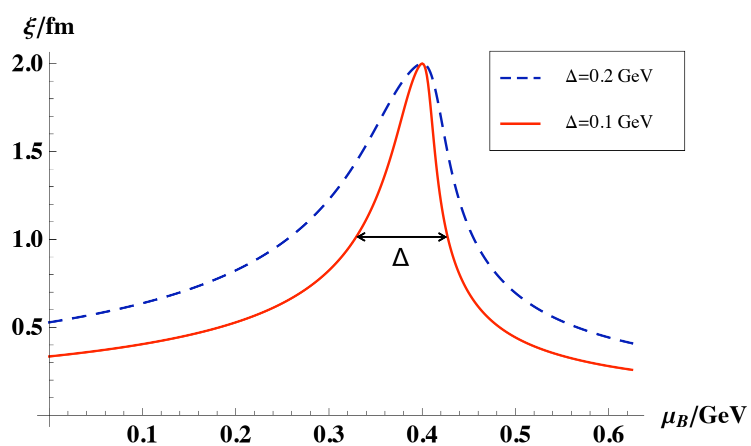
- **CP signature: Non-monotonic behavior, as a function of collision energy, of multiplicity cumulants**
- E.g. toy example

$$\xi(\mu_B) = \frac{2\ \text{fm}}{(1 + (\mu_B - 400)^2/W^2)^{1/3}}$$



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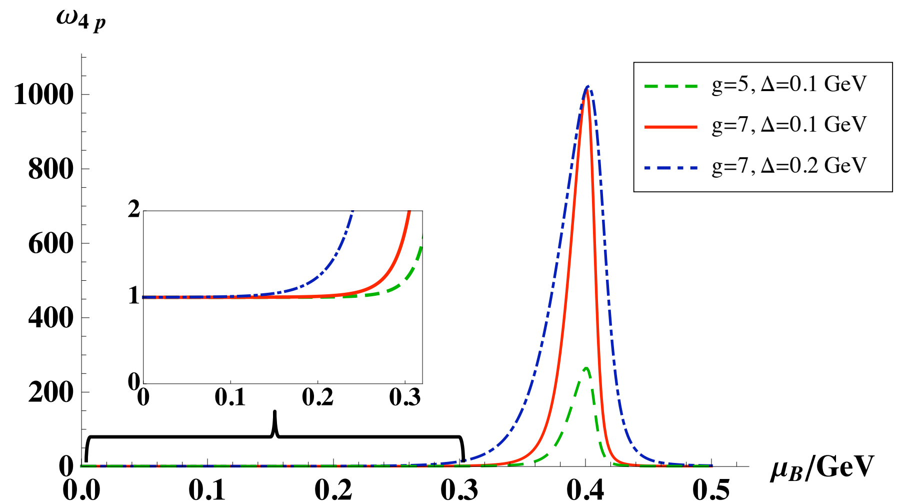
Multiplicity cumulants – example plots



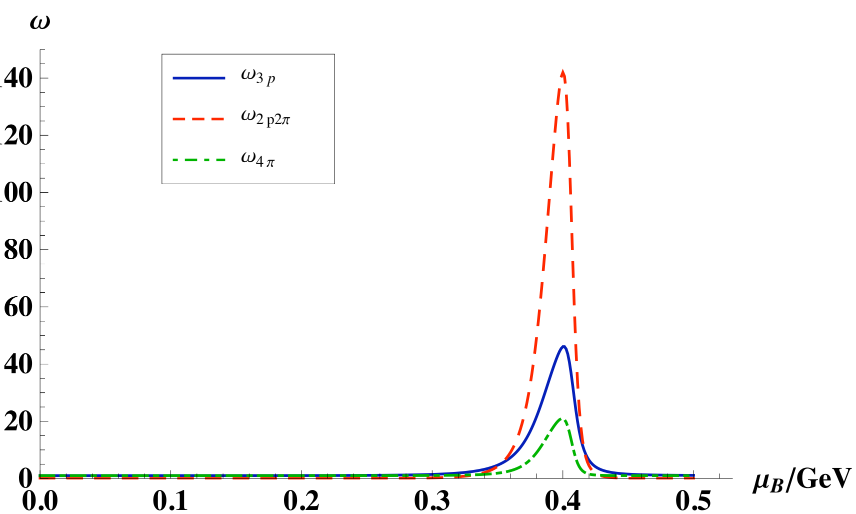
Parametrization (Cleymans et al 05):

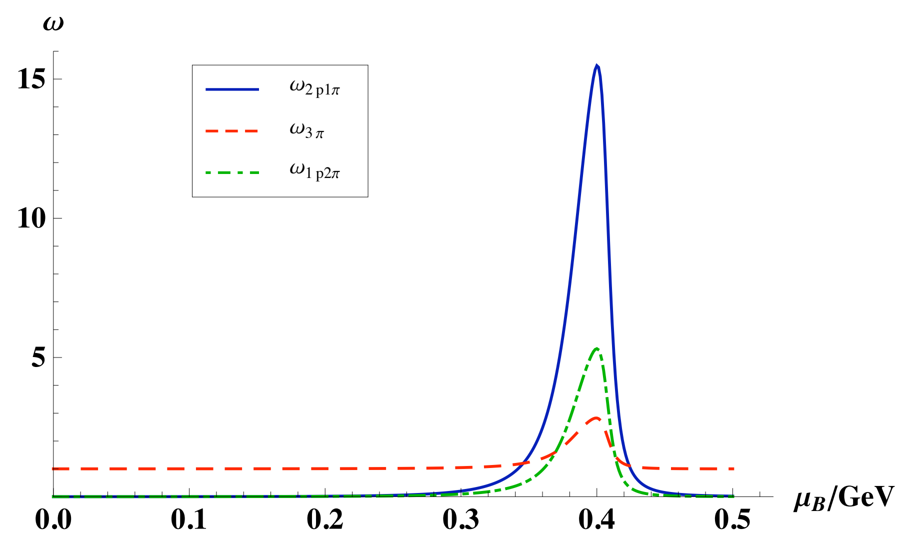
$$T(\mu_B) = a - b\mu_B^2 - c\mu_B^4$$

$$a = 0.166\ \text{GeV},\ \ \text{b} = 0.139\ \text{GeV}^{-1},\ \ \text{c} = 0.053\ \text{GeV}^{-3}$$

and using

$$\widetilde{\lambda}_3 = 6,\ \ \widetilde{\lambda}_4 = 22,\ \ G = 300\ \text{MeV},\ \ \text{g} = 7$$





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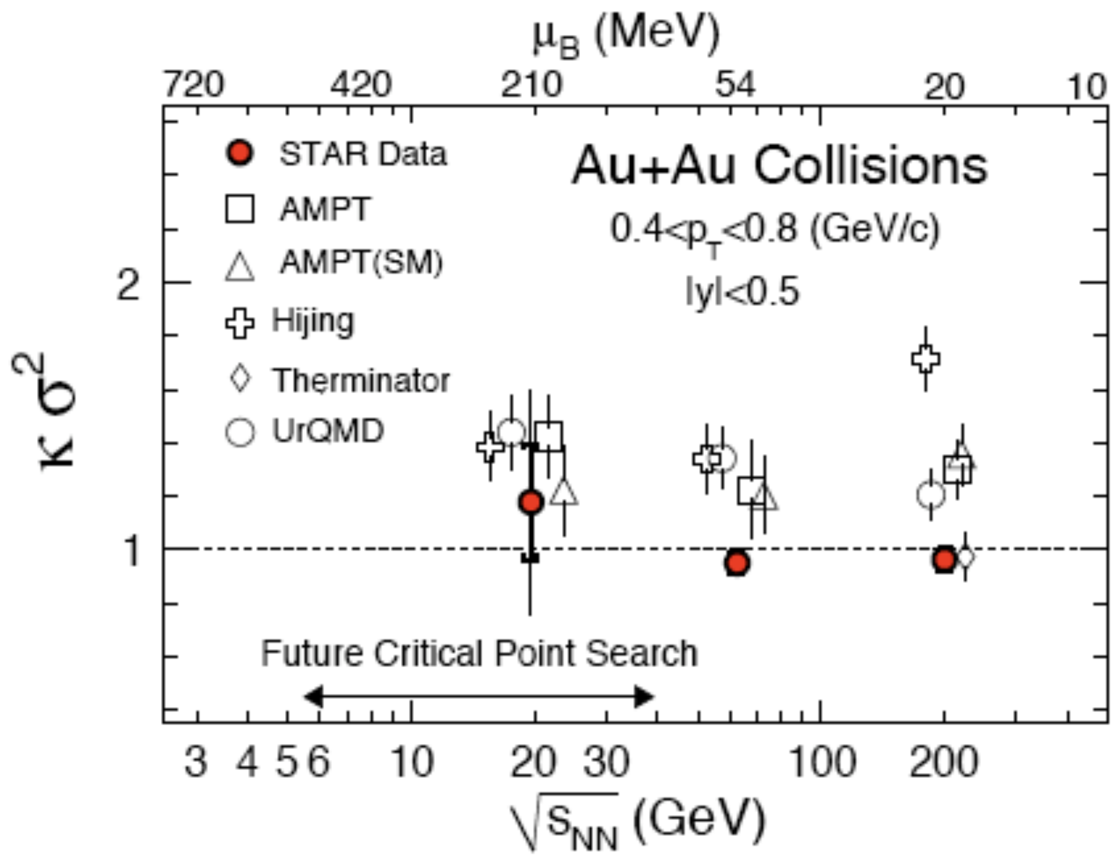
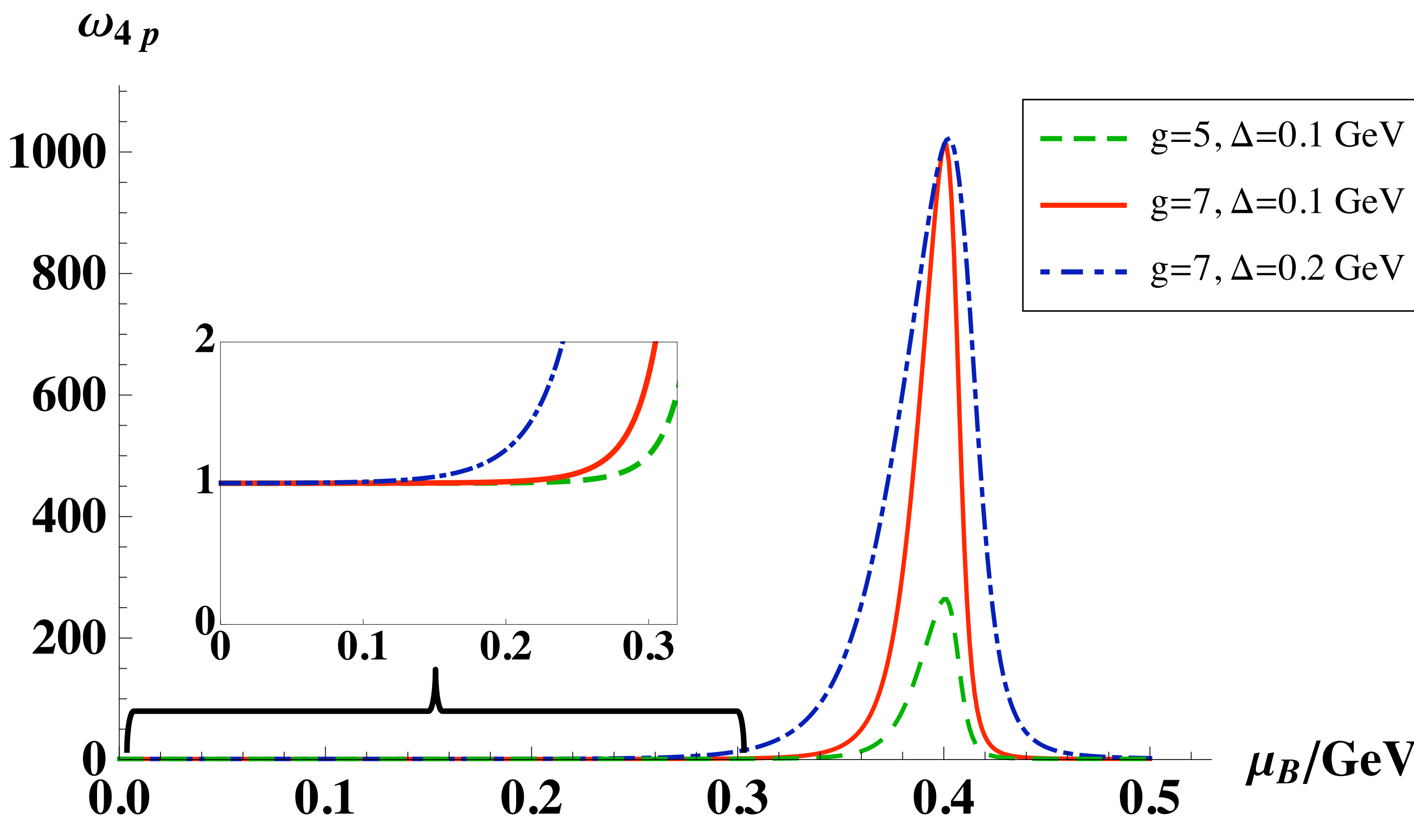
Data on net proton cumulants



where $\kappa\,\sigma^2 \equiv \frac{\kappa_4}{\kappa_2}$

(STAR Collaboration 2010)

Critical contribution to proton ω_4



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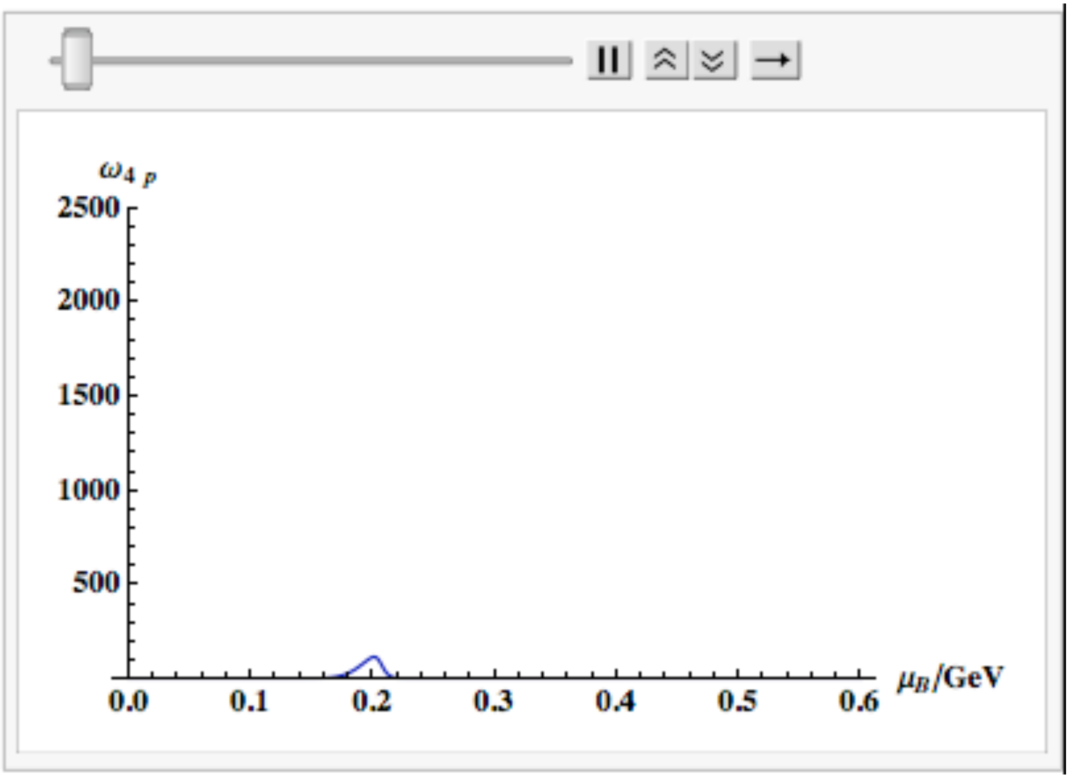
Multiplicity cumulants – movie

Changing the critical μ_B – the location of the CP:



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Outline

- Introduction
- Critical Contribution to Particle Multiplicity Fluctuations
- Ratios of Fluctuation Observables
- Summary

Uncertainties of parameters

- Cumulants depend on 5 non-universal parameters:

$$\kappa_{2\pi} \sim VT^{-1}G^2\xi^2 N_\pi^2,$$
$$\kappa_{3\pi} \sim VT^{-3/2}G^3\tilde{\lambda}_3\xi^{9/2}N_\pi^3,$$
$$\kappa_{4\pi} \sim VT^{-2}G^4(2\tilde{\lambda}_3^2 - \tilde{\lambda}_4)\xi^7 N_\pi^4$$

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⟹ hard to predict the critical contribution to cumulants

- By taking ratios of cumulants can cancel some parameter dependence

⟹ minimize observable uncertainties

Ratios of multiplicity cumulants

ratio	V	$n_p(\mu_B)$	g	G	$\tilde{\lambda}_3$	$\tilde{\lambda}_4'$	ξ
N_π	1	-	-	-	-	-	-
N_p	1	1	-	-	-	-	-
$\kappa_{ipj\pi}$	1	i	i	j	$\delta_{r,3}$	$\delta_{r,4}$	$\frac{5}{2}r-3$
$\omega_{ipj\pi}$	-	$i-\frac{i}{r}$	i	j	$\delta_{r,3}$	$\delta_{r,4}$	$\frac{5}{2}r-3$
$\kappa_{ipj\pi}N_\pi^{i-1}/N_p^i$	-	-	i	j	$\delta_{r,3}$	$\delta_{r,4}$	$\frac{5}{2}r-3$
$\kappa_{2p2\pi}N_\pi/\kappa_{4\pi}\kappa_{2p}$	-	-	-	-2	-	-	-2
$\kappa_{4p}N_\pi^2/\kappa_{4\pi}\kappa_{2p}^2$	-	-	-	-4	-	-	-4
$\kappa_{2p2\pi}N_p^2/\kappa_{4p}N_\pi^2$	-	-	-2	2	-	-	-
$\kappa_{3p1\pi}N_p/\kappa_{4p}N_\pi$	-	-	-1	1	-	-	-
$\kappa_{3p}N_p^{3/2}/\kappa_{2p}^{9/4}N_\pi^{1/4}$	-	-	$-3/2$	-	1	-	-
$\kappa_{2p}\kappa_{4p}/\kappa_{3p}^2$	-	-	-	-	-2	1	-
$\kappa_{3p}\kappa_{2\pi}^{3/2}/\kappa_{3\pi}\kappa_{2p}^{3/2}$	-	-	-	-	-	-	-
$\kappa_{4p}\kappa_{2\pi}^2/\kappa_{4\pi}\kappa_{2p}^2$	-	-	-	-	-	-	-
$\kappa_{2p2\pi}^2/\kappa_{4\pi}\kappa_{4p}$	-	-	-	-	-	-	-
$\kappa_{2p1\pi}/\kappa_{3\pi}^{2/3}\kappa_{3\pi}^{1/3}$	-	-	-	-	-	-	-

Ratios taken after subtracting Poisson and defined $\tilde{\lambda}_4' \equiv 2\tilde{\lambda}_3^2 - \tilde{\lambda}_4$

$r = i + j$

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Ratios of multiplicity cumulants

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$\kappa_{ipj\pi}$	1	i	i	j	$\delta_{r,3}$	$\delta_{r,4}$	$\frac{5}{2}r-3$
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$\kappa_{2p2\pi}N_p^2/\kappa_{4p}N_\pi^2$	-	-	-2	2	-	-	-
$\kappa_{3p1\pi}N_p/\kappa_{4p}N_\pi$	-	-	-1	1	-	-	-
$\kappa_{3p}N_p^{3/2}/\kappa_{2p}^{9/4}N_\pi^{1/4}$	-	-	$-3/2$	-	1	-	-
$\kappa_{2p}\kappa_{4p}/\kappa_{3p}^2$	-	-	-	-	-2	1	-
$\kappa_{3p}\kappa_{2\pi}^{3/2}/\kappa_{3\pi}\kappa_{2p}^{3/2}$	-	-	-	-	-	-	-
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$\kappa_{2p1\pi}/\kappa_{3\pi}^{2/3}\kappa_{3\pi}^{1/3}$	-	-	-	-	-	-	-

No parameter dependence

Ratios taken after subtracting Poisson and defined $\tilde{\lambda}'_4 \equiv 2\tilde{\lambda}_3^2 - \tilde{\lambda}_4$

$$r = i + j$$

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Parameter independent ratios

- Parameter and energy independent ratios:

$$r_1 = \frac{\mathrm{skewness_p}}{\mathrm{skewness}_\pi}, \quad r_2 = \frac{\mathrm{kurtosis_p}}{\mathrm{kurtosis}_\pi}, \quad r_3 = \frac{\kappa_{2p1\pi}}{\kappa_{3\pi}^{2/3}\kappa_{3\pi}^{1/3}}, \quad r_4 = \frac{\kappa_{2p2\pi}^2}{\kappa_{4\pi}\kappa_{4p}}$$

where $\mathrm{skewness} = \frac{\kappa_3}{\kappa_2^{3/2}}, \quad \mathrm{kurtosis} = \frac{\kappa_4}{\kappa_2^2}$

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- All equal to 1 if CP contribution dominates
- Poisson contribution: $r_1 = (N_\pi/N_p)^{1/2}, r_2 = N_\pi/N_p, \; r_3 = r_4 = 0$
- How to use these ratios:
  - If one sees peaks in the measured cumulants at some μ_B
  - Calculate these ratios around the peak
  - If equal to 1 ➡ Parameter independent way of verifying that the fluctuations you see are due to the CP

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Constraining parameters

- If CP found, can constrain parameters by measuring cumulant ratios near the CP

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- Parameters appear in certain combinations in the cumulants ⟹ can only constraint 4 independent (but not unique) combinations

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- Parameters appear in certain combinations in the cumulants ⟹ can only constraint 4 independent (but not unique) combinations
- For example, some choices are:

1. $G\,\xi$ using $\kappa_{2p2\pi}N_\pi/\kappa_{4\pi}\kappa_{2p}$ or $\kappa_{4p}N_\pi^2/\kappa_{4\pi}\kappa_{2p}^2,$
2. G/g using $\kappa_{2p2\pi}N_p^2/\kappa_{4p}N_\pi^2$ or $\kappa_{3p1\pi}N_p/\kappa_{4p}N_\pi,$
3. $\tilde{\lambda}_4'/\tilde{\lambda}_3^2$ using $\kappa_{2p}\kappa_{4p}/\kappa_{3p}^2,$
4. $\tilde{\lambda}_3^2/g^3$ using $\kappa_{3p}N_p^{3/2}/\kappa_{2p}^{9/4}N_\pi^{1/4}.$

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Summary

- We used particle multiplicity fluctuations as a probe to the location of the CP
- Higher cumulants of event-by-event distributions are more sensitive to critical fluctuations
- CP signature: Non-monotonic behavior, as a function of collision energy, of multiplicity cumulants
- Constructed cumulant ratios to identify the CP location with reduced parameter uncertainties
- If CP is found, showed how to use cumulant ratios to constraint the values of the non-universal parameters

Thank you!

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